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# Apportionment Strategies for the European Parliament

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**Abstract** A solution is proposed to the problem of assigning seats to new countries applying for membership of the European Parliament. This solution, simpler than many others, is obtained by weighting the populations and GDPs of all members. A strategy of optimization for each single country is also suggested.

*JEL Classifications* C71, D72
*Keywords* cooperative game theory, distribution of votes, European Parliament, power index, voting behaviour

## 1. Introduction

The apportionment of seats to incoming members to the European Parliament has always been a source of discussion and research (see for instance, Laruelle (1998)). It was discussed during the Nice Conference in 2000 and also during the accession negotiations with the applicant countries. Thanks to the Athens Accession Treaty 2003, already ten 'new' countries, joined the European Union in 2004.

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The emergence of a large number of countries joining the European Union involves a considerable increase in the total number of European Members of Parliament. Generally, the 'new' countries, having weaker economies than those of existing members, could influence decisions to the disadvantage of the current members.

The trend in the past was to take into account the size of population, to attempt to guarantee representation for major political parties of each country (e.g. Luxembourg) and avoid any reduction in the number of seats held by existing members. The democratic principle 'one man, one vote' – which means that only population should be considered – has given a negative result, because many countries (with small number of citizens) rested without votes. Already Turnovec (1997) and Mercik (1999) noticed the necessity to study the problem of how to divide the seats in the European Parliament, not only on the basis of the population but also taking into account other parameters.

Our proposition is to restructure the distribution of seats for all countries using a formula which takes into consideration both populations and Gross Domestic Products (GDPs). In such a way it is possible to give a simple and concrete answer to the problem under consideration. This idea will be presented in the next section. A related paradox and a number of theoretical aspects of coalition powers will be presented in sections 3 and 4. The optimum solutions for individual members of the new EU will be discussed in section 5. Some in-depth considerations will be given in section 6.

## 2. A method of seat distribution in EU Parliament

As it has been mentioned above, we propose a new method of distributing seats by means of a formula which takes into account both populations and GDPs. The most direct method consists in adequate weighting of these data using a convex linear combination. For instance, let populations and GDP percentages of the $i$-th country be shown by $P_i$ and $G_i$. Lets assume the weight for the population – $30\%$ and for GDP – $70\%$. In this case, the seat percentages $S_i$ of the $i$-th country will be $S_i = 0.3 \cdot P_i + 0.7 \cdot G_i$. In general, if $k$ is the weighting we wish to assign to the population $(0 \le k \le 1)$, the resulting seat percentages are $S_i = k \cdot P_i + (1-k) \cdot G_i$. To transform the seat percentages into the number of actual seats, a suitable rounding method can be used (for instance, Hondt's proportional system, or Hamilton's Greatest Divisors, or Gambarelli's (1999) minimax apportionment, and others).

Table 1 Seats in $0/100$ depending on weight $k$

| | *only ↓GDP* | | | | | *Values of k* | | | | | *only POP↓* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *0.0* | *0.1* | *0.2* | *0.3* | *0.4* | *0.5* | *0.6* | *0.7* | *0.8* | *0.9* | *1.0* |
| *GE* | **2435** | 2361 | 2288 | 2214 | 2141 | 2068 | 1994 | 1920 | 1847 | 1773 | 1700 |
| *FR* | **1638** | 1598 | 1557 | 1516 | 1475 | 1434 | 1393 | 1352 | 1312 | 1271 | 1230 |
| *UK* | **1589** | 1553 | 1517 | 1481 | 1445 | 1409 | 1374 | 1338 | 1302 | 1266 | 1230 |
| *IT* | **1342** | 1328 | 1314 | 1299 | 1285 | 1271 | 1257 | 1243 | 1228 | 1214 | 1200 |
| *SP* | 626 | 646 | 665 | 684 | 703 | 723 | 742 | 761 | 780 | 800 | **819** |
| *PL* | 180 | 242 | 305 | 367 | 430 | 492 | 554 | 617 | 679 | 742 | **804** |
| *RO* | 47 | 89 | 131 | 173 | 215 | 257 | 299 | 341 | 383 | 425 | **467** |
| *NE* | **428** | 418 | 408 | 398 | 388 | 378 | 368 | 358 | 348 | 338 | 328 |
| *GR* | 138 | 146 | 154 | 162 | 170 | 178 | 186 | 195 | 203 | 211 | **219** |
| *CR* | 63 | 78 | 93 | 108 | 123 | 138 | 154 | 169 | 184 | 199 | **214** |
| *BE* | **284** | 276 | 269 | 262 | 255 | 248 | 241 | 233 | 226 | 219 | 212 |
| *HU* | 54 | 69 | 85 | 101 | 116 | 132 | 148 | 163 | 179 | 194 | **210** |
| *PR* | 112 | 122 | 131 | 141 | 150 | 160 | 169 | 179 | 188 | 197 | **207** |
| *SW* | **269** | 261 | 252 | 244 | 235 | 227 | 218 | 210 | 201 | 192 | 184 |
| *BU* | 14 | 30 | 45 | 61 | 77 | 92 | 108 | 124 | 139 | 155 | **171** |
| *AU* | **239** | 232 | 225 | 218 | 211 | 203 | 196 | 189 | 182 | 175 | 168 |
| *SR* | 21 | 30 | 39 | 48 | 57 | 66 | 76 | 85 | 94 | 103 | **112** |
| *DE* | **197** | 188 | 179 | 171 | 162 | 153 | 145 | 136 | 127 | 119 | 110 |
| *FI* | **146** | 142 | 138 | 134 | 130 | 126 | 122 | 118 | 115 | 111 | 107 |
| *IR* | **96** | 94 | 93 | 91 | 89 | 87 | 85 | 83 | 82 | 80 | 78 |
| *LI* | 12 | 19 | 25 | 32 | 38 | 45 | 51 | 57 | 64 | 70 | **77** |
| *LA* | 7 | 11 | 16 | 20 | 25 | 29 | 33 | 38 | 42 | 46 | **51** |
| *SL* | 22 | 24 | 26 | 28 | 30 | 32 | 33 | 35 | 37 | 39 | **41** |
| *ES* | 6 | 8 | 11 | 13 | 16 | 18 | 20 | 23 | 25 | 28 | **30** |
| *CY* | 10 | 11 | 11 | 12 | 12 | 13 | 14 | 14 | 15 | 15 | **16** |
| *LU* | **21** | 20 | 18 | 17 | 16 | 15 | 14 | 12 | 11 | 10 | 9 |
| *MA* | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 8 | **8** |
| Total | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |

AU=Austria; BE=Belgium; BU=Bulgaria; CR=CzechRep.; CY=Cyprus; DE=Denmark; ES=Estonia; FI=Finland; FR=France; GE=Germany; GR=Greece; HU=Hungary; IR=Ireland; IT=Italy; LA=Latvia; LI=Lithuania; LU=Luxembourg; MA=Malta; NE=Netherlands; PL=Poland; PR=Portugal; RO=Romania; SL=Slovenia; SlovakRep.; SP=Spain; SR=; SW=Sweden; UK=UnitedKingdom.

Figure 1a Seats in $0/000$ of the top part of Table 1b, depending on $k$

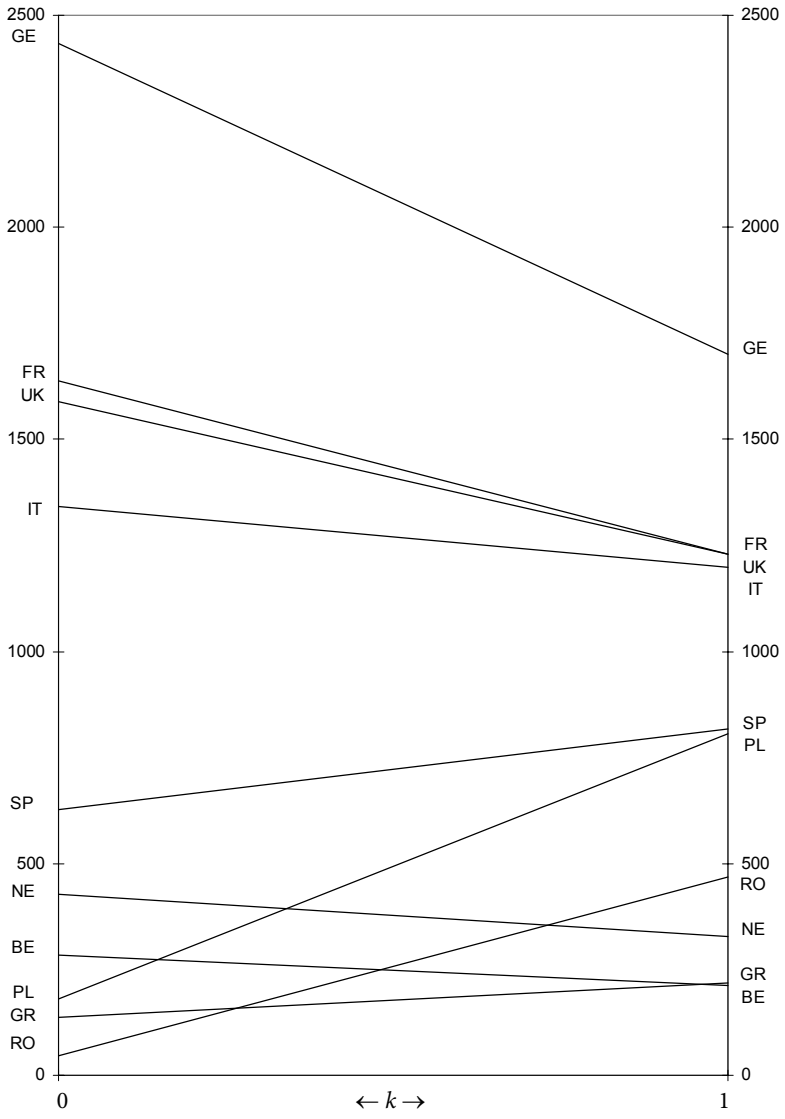


Figure 1b Seats in $0/000$ of bottom part of Table 1b, depending on $k$

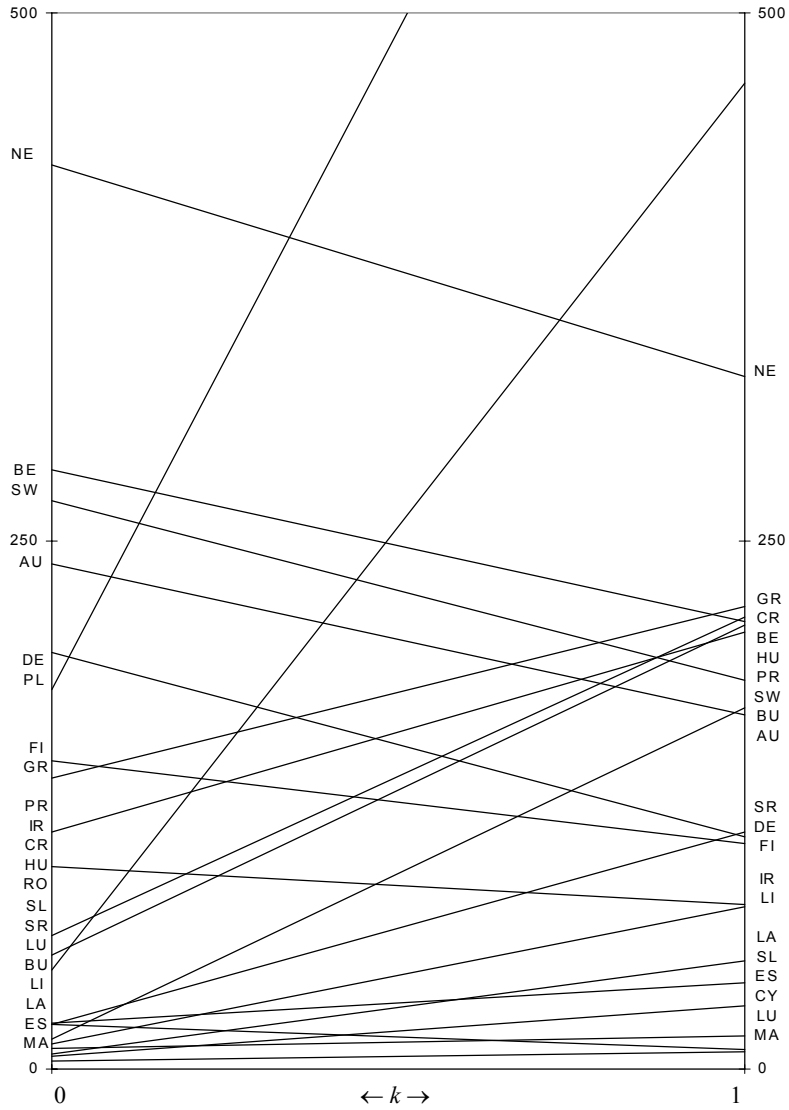


Table 1 presents the seat distribution, varying $k$ from 0 to 1 in 10% steps. The source of the first and last columns was IMF (2000). The GDP column was obtained by transforming the aggregate GDPs from local currencies into US dollars. For calculation of the other columns, the data used as a starting point had a greater number of decimal places and was later rounded using the Proportional System. The underlined figures show the maximum number of seats (in $0/100$ ) obtainable for each country.

The values in Table 1 are shown in Figures 1a and 1b in a continuous way. Figure 1a indicates countries with highest GDPs and only a few countries with low GDPs. Figure 1b shows all the other countries. As can be seen, each oblique segment represents one country. If a value $k$ is fixed on the horizontal axis and the vertical is drawn from this point, the points of intersection between the vertical and all the different segments indicate the seats (in $0/100$ ) to be allocated, depending on the chosen value of $k$. The value of the parameter $k$ strictly characterizes the seat distribution. In fact, if $k=0$, the seats are assigned proportionally on the basis of the countries' economic powers, without taking into account the size of population at all. And if $k=1$, vice versa. With $k=0$, the values of the first column of Table 1 are on the left vertical of Figure 1, whereas for $k=1$ the values of the last column are shown on the right vertical.

## 3. A surprise

Once this method of apportionment is accepted, only the question of fixing the value of $k$ will remain to weight the populations and GDPs, a discussion on this can be expected between countries with strong economies and those with weak economies. From an initial examination, it seems in the interests of countries with higher GDP percentages than their population percentages (Denmark, Finland, etc.) to have lower values of $k$ (preferably 0), as the respective segments decrease. Conversely, for countries with lower GDP percentages (Poland, Romania, etc.) it seems advantageous to have high values of $k$ (preferably 1), as the respective segments increase. However, this rule does not always apply, as will be illustrated with the following simple example.

Let us suppose that the Parliament consists of only three countries $A$, $B$ and $C$, with relative GDP percentages of $60$, $10$ and $30$, and population percentages of $40$, $40$ and $20$ (see Table 2 and Figure 2). The situation will be examined from $C$'s point of view. It seems preferable for $C$, having a GDP percentage $(=30)$ higher than its population percentage $(=20)$, to have a very low value of $k$: preferably 0. This gives country $C$ the maximum number of seats it could hope to attain $(30\%)$. In this case, however, country

$A$ would obtain $(60\%)$ of the seats, i.e. the majority in parliament. Then, country $C$ could not influence parliamentary decisions even if it formed a coalition with *B*. If, on the other hand, the maximum weight is given to the population percentage $(k=1)$, seat apportionment is equal to that of the populations $(40,40,20)$. Then the three countries have the same standing with respect to possible majority coalitions. To maintain its coalition strength, country $C$ would be better advised to give up a certain number of seats. But how many?

## 4. The power indices

Continuing with the example above, it can be seen that $C$ can avoid giving up a certain number of seats to maintain its coalition strength. In fact, if $k=0.6$, seat distribution is $(48,28,24)$ and $C$ is still able to form a winning coalition with $B$, i.e. the sum of seats of $C$ and B is sufficient to obtain the majority. If, on the other hand, $k=0.5$, then the seat distribution is $(50,25,25)$ and the coalition of $B$ and $C$ does not exceed $50\%$. The latter case is of a certain interest as, with $50\%$ of the seats, A does not have the majority on its own and, therefore, has to form a coalition with another country. It can be seen, thus, that the 'power' of a country is not proportional to the number of its seats when we take in consideration the capacity to form a winning coalition. This power of a country is called 'coalitional power'.

It should be noticed that apportionment of seats $(40,40,20)$ does not correspond with the vector of coalitional power $(0.4,0.4,0.2)$ but $(1/3,1/3,1/3)$ because none of the three countries has a majority on its own, and in order to obtain a majority, it has to form a coalition with another one; thus, from this point of view, they are of equal power. We have seen that the seat apportionment $(40,40,20)$ results in the division of coalitional power $(1/3,1/3,1/3)$ the apportionment $(60,10,30)$ results the division of coalitional power $(100\%,0,0)$.

What power can we assign to the apportionment of seats $(50,25,25)$? One method of evaluation is based on the concept of 'cruciality'. A member is said to be crucial for a coalition if this coalition is a majority with him and a minority without. In the case of $(50,25,25)$ country $B$ is crucial for just one coalition $(A,B)$, country $C$ for only one coalition $(A,C)$, whereas country $A$ is crucial for three coalitions: $(A,B)$, $(A,C)$, and $(A,B,C)$. The coalitional power of each member can be expressed in proportion to the number of coalitions to which it is crucial. In our case, this power is $3/5$ ths $(=60\%)$ for $A$ and $1/5$ th $(=20\%)$ for each of the other two. This power index is known as the 'Banzhaf-Coleman index' and it is one of the most applied

power indices; for further explanations, please see section 6.

Tables 2 and 3 and Figures 2 and 3 present a summary of the information given so far. It can be seen that the optimum value of $k$ for country $A$ is $0$, but the increase of $k$ to $0.4$ is not detrimental in terms of the absolute majority. With regard to $B$, the optimum value of $k$ is $0.6$ (which would give 28 seats) and its power would remain unchanged with a higher value of $k$. The optimal value of $k$ for $C$ is $0.6$. Such a value guarantees $C$ the maximum obtainable coalitional power $(33.3\%)$ and the maximum number of seats $(24)$.

An important addition must be made to complete this brief discussion of power indices. The observations assumed the formation of simple majorities (i.e. $>50\%$). If, on the one hand, we consider decisions with qualified majorities, the winning coalitions and, consequently, the power indices will obviously change. In decisions where there is a requirement for a majority of

Table 2 The seat distribution in the example of three countries' Parliament

| | *only ↓GDP* | | *Values of k* | | | | *only POP↓* |
|---|---|---|---|---|---|---|---|
| | *0.0* | … | *0.4* | *0.5* | *0.6* | … | *1.0* |
| *A* | 60 | … | 52 | 50 | 48 | … | 40 |
| *B* | 10 | … | 22 | 25 | 28 | … | 40 |
| *C* | 30 | … | 26 | 25 | 24 | … | 20 |
| *Total* | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Figure 2 The seat sharing in the example of three countries' Parliament, as a function of *k*

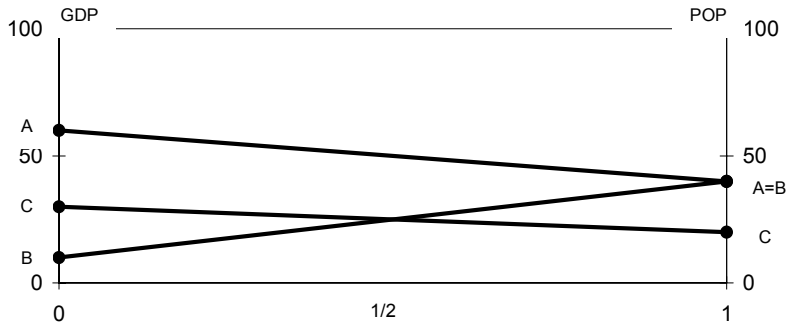
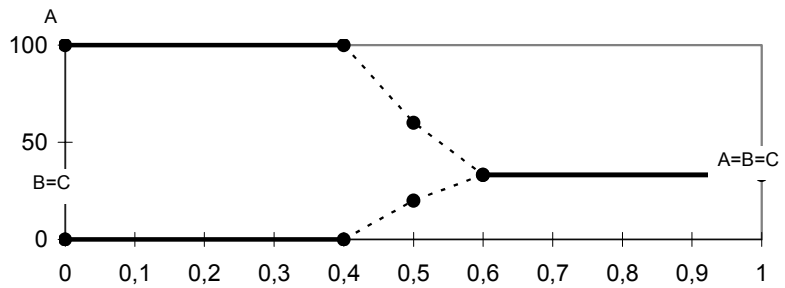


Table 3 The powers (%)corresponding to the seats from Table 2

| | *only ↓GDP* | | *Values of k* | | | | *only POP↓* |
|---|---|---|---|---|---|---|---|
| | *0.0* | … | *0.4* | *0.5* | *0.6* | … | *1.0* |
| *A* | 100 | … | 100 | 60 | 33.3 | … | 33.3 |
| *B* | 0 | … | 0 | 20 | 33.3 | … | 33.3 |
| *C* | 0 | … | 0 | 20 | 33.3 | … | 33.3 |
| *Total* | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Figure 3 The power (%) sharing in our example as a function of *k*




more than $3/4$ (i.e. $>75\%$), the only winning coalition is $(A,B,C)$ for the case of $(50,25,25)$. Then each party is crucial only once and the index is $(1/3,1/3,1/3)$. On the other hand, for the decisions requiring a majority of more than 2/3rds, the winning coalitions remain $(A,B)$, $(A,C)$ and $(A,B,C)$ and therefore the index remains $(3/5,1/5,1/5)$.

## 5. Optimum solutions for EU members

Table 4 gives the results of the calculations done on the apportionment of seats in Table 1. The calculations consider the threshold of a simple majority that is the prevalent rule in the European Parliament. For example, if we fix $k=0.3$, the seats will be distributed in accordance with the respective column of the Table 1 (Germany $22.14\%$, France $15.16\%$ and so on). For such division of seats, every country becomes crucial for a certain number of coalitions. The percentage of coalitions for which a given country is crucial,

Table 4 Powers ( 0/000 ) corresponding to the seats from Table 1

| | *only ↓GDP* | | | | | *Values of k* | | | | | *only POP↓* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *0.0* | *0.1* | *0.2* | *0.3* | *0.4* | *0.5* | *0.6* | *0.7* | *0.8* | *0.9* | *1.0* |
| *GE* | **2856** | 2724 | 2600 | 2483 | 2378 | 2284 | 2197 | 2115 | 2034 | 1946 | 1853 |
| *FR* | **1540** | 1504 | 1469 | 1438 | 1409 | 1383 | 1357 | 1332 | 1304 | 1271 | 1233 |
| *UK* | **1486** | 1453 | 1423 | 1397 | 1374 | 1354 | 1336 | 1316 | 1293 | 1265 | 1233 |
| *IT* | 1142 | 1144 | 1150 | 1159 | 1172 | 1185 | 1198 | 1207 | **1209** | 1207 | 1199 |
| *SP* | 606 | 639 | 671 | 701 | 728 | 751 | 767 | 776 | 782 | 787 | **793** |
| *PL* | 179 | 243 | 308 | 372 | 435 | 494 | 552 | 608 | 662 | 718 | **776** |
| *RO* | 47 | 89 | 132 | 173 | 214 | 252 | 290 | 327 | 365 | 406 | **452** |
| *NE* | **437** | 432 | 421 | 406 | 389 | 371 | 356 | 343 | 332 | 324 | 317 |
| *GR* | 137 | 146 | 155 | 162 | 169 | 174 | 180 | 186 | 193 | 202 | **211** |
| *CR* | 63 | 78 | 93 | 108 | 122 | 135 | 149 | 161 | 175 | 190 | **206** |
| *BE* | **283** | 278 | 271 | 264 | 254 | 243 | 233 | 223 | 215 | 209 | 205 |
| *HU* | 54 | 69 | 85 | 101 | 115 | 129 | 143 | 156 | 170 | 186 | **203** |
| *PR* | 111 | 122 | 132 | 141 | 149 | 156 | 163 | 171 | 179 | 188 | **200** |
| *SW* | **268** | 262 | 254 | 245 | 234 | 223 | 211 | 201 | 191 | 184 | 177 |
| *BU* | 14 | 30 | 45 | 61 | 76 | 90 | 104 | 118 | 132 | 148 | **165** |
| *AU* | **238** | 233 | 227 | 219 | 210 | 199 | 189 | 181 | 173 | 167 | 162 |
| *SR* | 21 | 30 | 39 | 48 | 56 | 64 | 73 | 81 | 89 | 98 | **108** |
| *DE* | **196** | 189 | 180 | 171 | 161 | 150 | 140 | 130 | 121 | 114 | 106 |
| *FI* | **145** | 142 | 139 | 134 | 129 | 123 | 118 | 113 | 109 | 106 | 103 |
| *IR* | **96** | 94 | 93 | 91 | 88 | 85 | 82 | 79 | 78 | 76 | 75 |
| *LI* | 12 | 19 | 25 | 32 | 38 | 44 | 49 | 54 | 61 | 67 | **74** |
| *LA* | 7 | 11 | 16 | 20 | 25 | 28 | 32 | 36 | 40 | 44 | **49** |
| *SL* | 22 | 24 | 26 | 28 | 30 | 31 | 32 | 33 | 35 | 37 | **39** |
| *ES* | 6 | 8 | 11 | 13 | 16 | 18 | 19 | 22 | 24 | 27 | **29** |
| *CY* | 10 | 11 | 11 | 12 | 12 | 13 | 13 | 13 | 14 | 14 | **15** |
| *LU* | **21** | 20 | 18 | 17 | 16 | 15 | 13 | 11 | 10 | 10 | 9 |
| *MA* | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 8 | **8** |
| Total | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |

i.e. its power index, is presented for each country in the column '*0.3*' of Table 4 (Germany 24.83% , France 14.69% , etc.).

In Table 4, the maximum values for each country in terms of power indices are underlined. Note that Italy, although it has a GDP percentage higher than its population percentage, does not have the maximum advantage when

only the GDP is taken into account (i.e. if $k=0$). Its maximum advantage is achieved with a balanced division $(k=0.8)$. It could be seen as a paradox; for a more detailed explanation of this result, please see section 4.

The figures in Table 4 provide information that may be helpful during the discussion concerning seat apportionment. The information tells each country how much the renouncement of every $1/10$ ths in the fixation of $k$ costs in terms of the coalitional power. For example, the change of $k$ from $0.7$ to $0.8$ costs Germany $0.81$ percentage points (from $21.15\%$ to $20.34\%$), while the change of $k$ from $0.8$ to $0.9$ costs the same country $0.88$ percentage points (from $20.34\%$ to $19.46\%$). In the case of Table 1, the decrements of the percentages seats are always constant.

## 6. Some in-depth considerations

Let us follow some considerations dedicated to those readers who will want to explore in depth some of the arguments dealt with above.

### 6.1 *On power indices*

The power index used here was in the past known as the 'Normalized Banzhaf index' [Banzhaf (1965)] and later as 'Banzhaf-Coleman Index', thanks to the contribution of Coleman (1971). There are other indices that use crucialities, but they take it into account differently and they are based on particular bargaining models. For instance the index of Shapley-Shubik (1954) (based on the Shapley (1953) value of a game), the Holler (1978) index, and the Nucleolus [Schmeidler (1969)]. Further information on the matter can be found in Owen (1995), Gambarelli and Owen (2000 and 2004), Gambarelli (1994 and 1999). Some algorithms for the computing of these indices are known [see for instance, Gambarelli (1990 and 1996)]. A modification of the program quoted in Bilbao (2000) and Bilbao et al. (2000) has been used for the computations of Table 4.

Various studies on the applications of power indices to the European Parliament and the Council of Ministers structure were developed in the past century (see for instance the simulations of Gambarelli and Holubiec (1990)). More recently, the Nice conference stimulated the emergence of very important contributions; we quote in particular Bilbao (2000 and 2001), Felsenthal and Machover (1998, 2001 and 2003). Further applications based on the above quoted papers as well as Affuso and Brams (1985), Berg, Lane and Maeland (1996), Brams (1976), Gambarelli and Owen (1994), Garrett and

Tsebelis (1999), Herne and Nurmi (1993), Holler and Widgrén (1999), Laruelle, Martinez and Valenciano (2003), Laruelle and Widgrén (1998), Owen and Shapley (1989), Turnovec (1996), Widgrén (1996) are under consideration.

### 6.2 *On overtaking*

In the cases examined here we have varied $k$ in steps of $1/10$ th, but in general this parameter can vary in the continuous interval $[0,1]$. The values of $k$ corresponding to the intersections of segments are of particular interest. In fact, each of these intersections represents a situation of equality of voting power of two countries. In formal terms, let $G_i$ and $P_i$ be the GDP and the populations of the $i$-th country, respectively. An intersection between the segments representing countries $i$ and $j$ falls within the interval $(0,1)$ whenever

$$(G_i < G_j \text{ and } P_i > P_j) \text{ or } (G_i > G_j \text{ and } P_i < P_j).$$

The corresponding value of $k_{ij}$ is:

$$k_{ij} = (G_j - G_i)/[(P_i - P_j) + (G_j - G_i)]$$

### 6.3 *On optimum weight intervals*

In the evaluations made in section 4 with regard to the optimum values of $k$ for each country, we took only variations of $k$ with $0.1$ steps into account. This led to optimum intervals $[0, 0.4]$ for $A$ and $[0.6, 1]$ for $B$ and $C$. If we now vary $k$ in a continuum, we discover an interval of values even more advantageous for $C$. If there are $100$ seats to be assigned and the Hondt Proportional System is used for rounding, then each weight $k$ satisfying the inequalities $0.52 < k < 0.55$ leads to $(49,26,25)$, which is the optimum distribution of seats for $C$. The optimum values of $k$, however, remain unchanged for the other two countries. In more theoretical terms, we can define 'optimum weight interval of the $i$-th Country' as the variability interval $k_i$ which guarantees this country the maximum power index, and with equal power index, the maximum number of seats. In our example it is possible to check that the optimum intervals for the three countries are as described in Table 5. Further studies on the behavior of optimum weight intervals could be carried out, based on Gambarelli (1983), Freixas and Gambarelli (1997) and Saari and Sieberg (2000).

Table 5 Optimum weight intervals for the example in Table 2

| | *Optimal interval* | *Seats* | *Power indices* |
|---|---|---|---|
| *A* | [0, 0.025) | 60, 10, 30 | 1, 0, 0 |
| *B* | (0.98, 1] | 40, 40, 20 | 1/3, 1/3, 1/3 |
| *C* | (0.52, 0.55) | 49, 26, 25 | 1/3, 1/3, 1/3 |

## 7. Concluding remarks

This paper presents a proposal on how to divide seats when countries with weaker economies enter the European Union. Our proposal, considering the threshold of a simple majority, is simpler than other proposals being studied currently. This method takes into account both populations and GDP. As regards the weights to be assigned to these two components, a paradox relating to Italy was discussed. Such paradox is due to the fact that although Italy had the maximal seat percentage taking into account only GDP this country did not receive the maximal power in this situation (for more particulars see section 5). Moreover, reference information for the discussion on the determination of such weights was provided for each country. Similar paradoxes, which, however, are not presented in this paper, were also carried out with others percentages. It is a remarkable paradox relating to the Netherlands and to Ireland obtained with the percentage of 62% .

The majority system of the European Parliament foresees majorities even distinct from the simple majority, but the authors take 50% threshold into consideration because it seems to be the most intuitive and the choice does not have any influence on the argument presented.

A practical application of the method described above must include a proper rule for the calculation of GDP, both in the present situation and in the future updates which might be necessary, such as for example new elections, new entries, or important changes in the percentages.

The method proposed in this paper can be a starting point for some in-depth considerations and other studies. The first necessary consideration is for the population and GDP, here considered as national items. From the high correlation existing between these two quantities, some of the arguments which hold for national population hold also for national GDP weights. It is interesting to extend the above results considering the couple – population and GDP – also in a different way which can best reflect international nature of the European Parliament where many factions of the voting body are international. In fact, in the Chamber, the members sit in political

groups (PPE-DE, PSE, ELDR and so on), not in national delegations. Other consideration could be done through the analysis of the decision power of particular parliaments in the light of the studies of Owen on the a priori coalitions [Owen (1977, 1982), Carreras and Owen (1988)] and on the optimal location of candidates in ideological space [Owen and Shapley (1989)]. Last but not least consideration is the comparison of above results with the results concerning to the others power indices like those of Shapley-Shubik and Holler.

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